

Oscillation characteristics of active and sterile neutrinos and neutrino anomalies at short distances

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A generalized phenomenological $(3 + 2 + 1)$ -model with three active and three sterile neutrinos is considered for the calculation of the neutrino oscillation characteristics at normal mass hierarchy of active neutrinos and significant splitting between the mass of one sterile neutrino and the masses of other two sterile neutrinos. A new parametrization and a certain form of the general neutrino mixing matrix for active and sterile neutrinos are suggested taking into account the possible violation of CP invariance in the lepton sector. The test values of the neutrino masses and the mixing parameters are chosen. The transition probabilities for the different flavors of neutrino, which can be transformed into each other, are calculated, and the graphical dependences are obtained for the disappearance probability of muon neutrino/antineutrino and appearance probability of electron neutrino/antineutrino in the muon neutrino/antineutrino jet as a function of distance and other model parameters at their acceptable values and at neutrino energies not higher than 50 MeV, as well as a function of the ratio of the distance to the neutrino energy. It is shown that in the case of the mixing matrix of a definite type (of the a_2 type) between active and sterile neutrinos the explanation of the acceleration anomaly on small distances in the neutrino data (LSND anomaly) is possible, as well as the reactor and gallium anomalies. The theoretical results obtained can be used for the interpretation and prediction of results of the ground-based neutrino experiments on searching for the sterile neutrinos, and also for the analysis of certain astrophysical observation data.

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1. INTRODUCTION

An one of the most important problems of the modern neutrino physics is the problem of light sterile neutrinos, which is related to neutrino- and antineutrino-flux anomalies observed at short distances in a number of ground-based experiments [1–4]. The presence of such anomalies, if it will be confirmed at a sufficiently high reliability level, is obviously beyond the Standard Model (SM), as well as the Minimally Extended Standard Model (MESM) with three active neutrinos of different mass, since oscillations of only three known active neutrinos cannot explain these anomalies. Sterile neutrinos are new particles that do not interact with the SM particles via the exchange by photons, W and Z bosons, and naturally gluons [5]. The scale of sterile neutrino masses that is required for interpretation of the anomalies mentioned above is about 1 eV.

In principle, sterile-neutrino mass values may belong to a broad energy range from 10^{-5} eV to 10^{15} GeV [6, 7]. It is convenient to break down this range in such a way as to refer sterile neutrinos of mass below 0.1 eV to a class of ultralight sterile neutrinos, neutrinos of mass between 0.1

and 100 eV to a class of light sterile neutrinos, neutrinos of mass between 100 eV and 10 GeV to a class of heavy sterile neutrinos, and neutrinos of mass above 10 GeV to a class of superheavy sterile neutrinos. Intensive experimental and theoretical studies are being currently performed in order to solve the problem of light sterile neutrinos, which are involved to interpret the accelerator anomaly observed in the LSND/MiniBooNe experiments [8–10], as well as the reactor [11, 12] and gallium [13, 14] anomalies, and some astrophysical data [15]. It is expected that experimental data making it possible to confirm or disprove the existence of the aforementioned anomalies will be obtained in the near future (see, for example, Refs. [1, 5, 16–18]).

Since the existence of sterile neutrinos is beyond the MESM framework, different phenomenological models with one, two, or three sterile neutrinos were proposed [1–5, 19, 20]. In principle, the number of sterile neutrinos may be arbitrary as long as this is compatible with the experimental data. If, however, to take into account the possible existence of the left-right symmetry of weak interactions and to associate sterile neutrinos with right-handed neutrinos, which are neutral with respect to $SU(2)_L$ -weak interactions, then the number of sterile neutrinos should be equal to three, that is, it is necessary to consider a $(3 + 3)$ -model with three active and three sterile neutrinos [19, 20]. As one such model, there is the $(3 + 1 + 2)$ -model (or $(3 + 2 + 1)$ -model) with three

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sterile neutrinos [21, 22], where two of them are approximately degenerate in mass, while the third one has a mass that may differ markedly from the masses of the other two sterile neutrinos. The $(3+2+1)$ -model involving two ultralight sterile neutrinos and one light sterile neutrino was used to evaluate the mass characteristics of both active and sterile neutrinos [21] and to calculate the appearance and survival probabilities for active and sterile neutrinos in the Sun with allowance made for the coherent neutrino scattering in matter [22]. An enhancement of the yield of three sterile neutrinos in high-density media as the neutron-to-proton number ratio approaches two was considered in Ref. [23]. This effect may have an influence on the formation of neutrino fluxes in supernovae.

It is well known that there are difficulties in models with three active and three sterile neutrinos in matching the whole number of neutrinos with data coming from the cosmological observations (see, for example, Refs. [24, 25]). At the present time, the most popular models are phenomenological models with one or two sterile neutrinos, that is the so-called $(3+1)$ - and $(3+2)$ -models [1–5]. The basic argument for this choice is that, according to currently prevalent ideas, one or two sterile neutrinos are sufficient for explaining the experimentally observed anomalies. Moreover, the most recent data obtained on the basis of standard cosmological models from observations of the cosmic microwave background restrict the number of new relativistic neutrinos up to one [24, 25]. Despite this, the present study is devoted to considering a phenomenological model that involves three active and three sterile neutrinos. There are several reasons for this choice. First, this is the aforementioned principle of left-right symmetry, which is possibly restored at a scale that exceeds the scale of spontaneous breaking of electroweak symmetry (of the order of 1 TeV). Second, the form chosen in Sec. 3 for mixing matrix of both active and sterile neutrinos dictates the introducing namely three sterile neutrinos, provided that the mixing matrix satisfies the correct unitarity condition. And finally, one cannot exclude that nonstandard cosmological models may prove to be viable, in framework of which, despite the presence of three sterile neutrinos, their effect is reduced or suppressed in data from cosmological observations (see, for example, Refs. [26, 27]).

The paper is organized as follows. In Sec. 2, we present the experimental data on the oscillation characteristics of active neutrinos and describe the experimental indi-

cations of possible existence of the anomalies that go beyond the scope of the Minimally Extended Standard Model with three active massive neutrinos. In Sec. 3, the fundamentals of a generalized neutrino model with three active and three sterile neutrinos are discussed with allowance for the results reported in Refs. [21, 22]. In Sec. 4, we consider a way to determine the oscillation properties of both active and sterile neutrinos on the basis of equations that describe the propagation amplitudes for neutrino in a vacuum. In the same section, we also present exact analytical expressions for determining the appearance and disappearance (survival) probabilities for neutrinos of specific flavor, which are particular cases of the general expressions from Ref. [28] for the model under consideration. The results of our numerical calculations of the oscillation properties of neutrinos at test values of the neutrino masses and mixing parameters, which are performed with allowance for the sterile-neutrino contributions, are given in Sec. 5. We obtain the appearance probabilities for electron neutrinos/antineutrinos in beams of muon neutrinos/antineutrinos versus the distance from the neutrino source and versus the ratio of this distance to the neutrino energy, and also we obtain the disappearance probabilities for muon neutrinos/antineutrinos at model-parameter values typical for the accelerator anomaly. In the concluding Sec. 6, we discuss the paper results, which may be of use in interpreting and predicting the results of experiments devoted to searching for the effects associated with sterile neutrinos, and also in analyzing some astrophysical data.

2. OSCILLATION CHARACTERISTICS OF ACTIVE NEUTRINOS AND ANOMALIES IN NEUTRINO DATA

As is well known, oscillations of solar, atmospheric, reactor, and accelerator neutrinos may be explained by the mixing of different neutrino mass eigenstates. This means that the flavor states of active neutrinos (a) are a mixture of at least three neutrino mass eigenstates, and vice versa. The mixing of neutrino states is described in terms of the Pontecorvo–Maki–Nakagawa–Sakata matrix $U_{PMNS} \equiv U \equiv VP$ as

$$\psi_L^a = U_i^a \psi_L^i, \quad (1)$$

where $\psi_L^{a,i}$ are left-handed chiral fields of flavor or massive neutrinos, respectively, and $a = \{e, \mu, \tau\}$, $i = \{1, 2, 3\}$. For three active neutrinos, the matrix V can be expressed in terms of a standard parametrization [29] as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, $\delta \equiv \delta_{CP}$ is the phase associated with Dirac's CP violation in the lepton sector,

and $P = \text{diag}\{1, e^{i\alpha}, e^{i\beta}\}$, with $\alpha \equiv \alpha_{CP}$ and $\beta \equiv \beta_{CP}$

the phases associated with Majorana's CP violation.

In general, a unitary $n \times n$ matrix is determined by n^2 real-valued parameters, for which one can take $n(n-1)/2$ angles and $n(n+1)/2$ phases. With allowance for the structure of the SM electroweak Lagrangian, which includes currents composed of quark, charged-lepton, and neutrino fields, it is possible to exclude $2n-1$ phases in the case of Dirac neutrino fields. But in the case where the neutrino fields belong to the Majorana type, we can exclude only n phases associated with Dirac charged leptons. In view of this, a $n \times n$ mixing matrix is determined either by $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases if neutrinos are Dirac particles or by $n(n-1)/2$ angles and $n(n-1)/2$ phases if neutrinos would be Majorana particles [30]. Thus, for three active Dirac neutrinos it is necessary to determine three mixing angles θ_{12} , θ_{13} , θ_{23} and one mixing phase (CP phase δ_{CP}) in order to specify the mixing matrix U_{PMNS} , while for three active Majorana neutrinos one needs to specify the analogous three mixing angles (θ_{12} , θ_{13} , θ_{23}) and three CP phases δ_{CP} , α_{CP} , and β_{CP} .

However, oscillation experiments with atmospheric, solar, reactor, or accelerator neutrinos give no possibilities to measure the Majorana CP phases and to attribute neutrinos to the Majorana or Dirac particle type. Nevertheless, experimental results concerning neutrino oscillations suggest violation of conservation laws of the lepton numbers L_e , L_μ , and L_τ and, because of nonzero values of two oscillation parameters Δm_{12}^2 and Δm_{13}^2 (where $\Delta m_{ij}^2 = m_i^2 - m_j^2$), the existence of at least two nonzero active-neutrino masses that are not equal to each other.

Now we present the values for the neutrino mixing angles and mass-squared differences with standard uncertainties at an 1σ level, which specify three-flavor oscillations of light active neutrinos in a vacuum, which were obtained from a global analysis of the most recent high-precision measurements of neutrino oscillation parameters [31]. Specifically, they are

$$\sin^2 \theta_{12} = 0.308_{-0.017}^{+0.017}, \quad (3a)$$

$$\sin^2 \theta_{23} = \begin{cases} \text{NH} : 0.437_{-0.023}^{+0.033} \\ \text{IH} : 0.455_{-0.031}^{+0.039} \end{cases}, \quad (3b)$$

$$\sin^2 \theta_{13} = \begin{cases} \text{NH} : 0.0234_{-0.0019}^{+0.0020} \\ \text{IH} : 0.0240_{-0.0022}^{+0.0019} \end{cases}, \quad (3c)$$

$$\Delta m_{21}^2/10^{-5}\text{eV}^2 = 7.54_{-0.22}^{+0.26}, \quad (3d)$$

$$\Delta m_{31}^2/10^{-3}\text{eV}^2 = \begin{cases} \text{NH} : 2.43_{-0.06}^{+0.06} \\ \text{IH} : -2.38_{-0.06}^{+0.06} \end{cases}. \quad (3e)$$

The CP phases α_{CP} , β_{CP} , and δ_{CP} and the neutrino mass scale are still not known at the present time. Since only the absolute value of the oscillation characteristic $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$ is known, the absolute values of the neutrino masses can be ordered by two ways, namely, a) $m_1 < m_2 < m_3$ and b) $m_3 < m_1 < m_2$, that is, it may be either a normal hierarchy [NH, case (a)] or an inverse hierarchy [IH, case (b)] in the neutrino-mass spectrum.

Along with the values presented in Eqs. (3) for the oscillation characteristics of the neutrinos, indications that the neutrino fluxes data for some processes have anomalies that cannot be explained by oscillations of only active (that is, electron, muon, and tau) neutrinos and antineutrinos have existed for a rather long time. These anomalies include the LSND (accelerator) anomaly [8–10] and the reactor [11, 12] and gallium (calibration) [13, 14] anomalies, which can be explained by the existence of one or two extra neutrinos that do not interact with other SM particles, that is, sterile neutrinos. The characteristic scale of sterile-neutrino masses that is required for describing the aforementioned anomalies is about 1 eV. These anomalies manifest themselves at short distances (more precisely, at distances L such that the numerical values of the parameter $\Delta m^2 L/E$, where E is the neutrino energy, are about unity). The experiments at such distances are referred to as short-baseline (SBL) experiments. The possible excess of the electron antineutrino fraction in beams of muon antineutrinos in relation to what is expected within the MESM framework is referred to as the LSND anomaly (sometimes accelerator anomaly). Similar results were also observed in the MiniBooNE experiments for electron neutrinos and antineutrinos (see, however, Ref. [3]). A deficit of reactor electron antineutrinos at short distances was called as the reactor anomaly, while a deficit of electron neutrinos from a radioactive source in the calibration of the detectors for the SAGE and GALLEX experiments is usually referred to as the gallium (calibration) anomaly. In other words, data on the anomalies concern both the appearance of electron neutrinos or antineutrinos in the fluxes of muon neutrinos or antineutrinos, respectively, and the disappearance of electron or muon neutrinos or antineutrinos. It should be emphasized that these three types of anomalies observed to date in the neutrino fluxes data suggest the existence of light sterile neutrinos.

It should be noted that recent data coming from astrophysical observations and concerning the formation of galaxies and their clusters can be explained by the existence of light or heavy sterile neutrinos. In astrophysics, such sterile neutrinos are real candidates for dark-matter particles [15, 32, 33]. At the present time, multicomponent dark-matter models are frequently used to describe dark-matter-dominated objects such as some galaxies and galaxy clusters [15]. More details on the possible existence of sterile neutrinos, their properties, and their relation to still unknown dark-matter particles can be found in numerous papers (see, for example, Refs. [1, 15, 33–35]).

3. FUNDAMENTALS OF THE PHENOMENOLOGICAL $(3+2+1)$ -MODEL OF ACTIVE AND STERILE NEUTRINOS

Below, we consider a phenomenological $(3+2+1)$ neutrino model, which is used to include three sterile neutrinos in the formalism of theory of weak interactions. This

model can be employed both to obtain, with allowance made for sterile neutrinos, the properties of neutrino fluxes studied in ground-based SBL experiments and to analyze some astrophysical data, including the data on neutrino radiation from supernovae [21–23]. The model under consideration includes three active neutrinos and three sterile neutrinos, such that two sterile neutrinos are approximately degenerate in mass, while the mass of the third sterile neutrino may differ significantly from the masses of the other two sterile ones.

Taking into account the results obtained in Refs. [21, 22], let us consider the neutrino-mixing formalism for the case of three active and three sterile neutrinos. We will use the indices x, y , and z to distinguish between the different sterile-neutrino types (flavors) and the indices $1', 2'$, and $3'$ to distinguish between the extra massive states. Further, we will denote by s the set of indices x, y , and z and by i' the set of indices $1', 2'$, and $3'$. A general 6×6 mixing matrix \tilde{U} can then be expressed in terms of the 3×3 matrices S, T, V , and W as

$$\begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix} = \begin{pmatrix} S & T \\ V & W \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_{i'} \end{pmatrix}. \quad (4)$$

The neutrino masses will be specified by means of the set $\{m\} = \{m_i, m_{i'}\}$ ordered normally for $\{m_i\}$ as $\{m_1, m_2, m_3\}$ and inversely for $\{m_{i'}\}$ as $\{m_{3'}, m_{2'}, m_{1'}\}$. For the unitary 6×6 matrix \tilde{U} , we will consider only some particular cases rather than the most general form, involving additional physical assumptions. On the basis of data available from astrophysical and laboratory observations, we assume that the mixing between active and sterile neutrinos is small. In addition, the basis for sterile-neutrino mass eigenstates will be constructed from states for which the matrix W is approximately equal to the identity matrix. In doing this, we restrict ourselves to a diagonal matrix of the form $W = \tilde{\kappa}I$, where I is the identity matrix and $\tilde{\kappa}$ is a complex-valued parameter whose modulus is close to unity, and for which we employ the form $\tilde{\kappa} = \kappa \exp(i\phi)$. The matrix S will be represented in the form

$$S = U_{PMNS} + \Delta U_{PMNS}, \quad (5)$$

where the matrix ΔU_{PMNS} , as well as the matrix T in Eq. (4) should be small towards U_{PMNS} . In order to the quantitative estimations of both the mixing between active and sterile neutrinos and the sterile-neutrino-induced corrections to mixing between active neutrinos to be convenient, we assume that

$$\Delta U_{PMNS} = -\epsilon U_{PMNS}, \quad (6)$$

where ϵ is a small quantity of the form $\epsilon = 1 - \kappa$. The matrix S then has the form $S = \kappa U_{PMNS}$, where U_{PMNS} is the known unitary 3×3 matrix of active-neutrino mixing ($U_{PMNS} U_{PMNS}^\dagger = I$).

Thus, upon the respective normalization that was chosen, the active neutrinos are mixed, as it should be in the

MESM, according to the Pontecorvo–Maki–Nakagawa–Sakata matrix. Taking into account that the mixing between active and sterile neutrinos is small and is determined by the small parameter ϵ , we choose the matrix T as $T = \sqrt{1 - \kappa^2} a$, where a is an arbitrary unitary 3×3 matrix, so that $aa^\dagger = I$. The 6×6 mixing matrix \tilde{U} , which can be written now in the form of

$$\begin{aligned} \tilde{U} &= \begin{pmatrix} \kappa U_{PMNS} & T \\ -e^{i\phi} T^\dagger U_{PMNS} & \kappa e^{i\phi} I \end{pmatrix} \\ &\equiv \begin{pmatrix} \kappa U_{PMNS} & \sqrt{1 - \kappa^2} a \\ -e^{i\phi} \sqrt{1 - \kappa^2} a^\dagger U_{PMNS} & \kappa e^{i\phi} I \end{pmatrix}, \end{aligned} \quad (7)$$

is then strictly unitary. In Refs. [21, 22], use was made of an approximately unitary mixing matrix of the form analogous to Eq. (7) at $\phi = 0$, where, however, the condition requiring the conservation of normalization for the sterile-neutrino states was not taken into account (corrections associated with a nonunitarity of the mixing matrix were discussed, for example, in Ref. [36]). In the present study, we take $\phi = \pi/4$ for the test value of the unknown phase ϕ .

Although the value of the CP phase δ_{CP} has not yet been established experimentally, estimates for it were obtained in a number of studies (see, for example, Refs. [31, 37, 38]) for the case of a normal hierarchy for the active-neutrino mass spectrum, namely, $\sin \delta_{CP} < 0$ and $\delta_{CP} \approx -\pi/2$. A normal hierarchy also becomes more preferable upon taking into account the constraints on the sum of the neutrino masses from data based on the cosmological observations [39]. In our further numerical calculations, we therefore restrict ourselves to the NH case, setting $\delta_{CP} = -\pi/2$.

In the present study, we consider three particular cases for the form of the matrix a , which specifies mixing between active and sterile neutrinos. They will be denoted by a_1, a_2 , and a_3 . For the first two cases, we choose these matrices in the form

$$a_1 = \begin{pmatrix} e^{-i\chi_1} & 0 & 0 \\ 0 & \cos \eta_1 & \sin \eta_1 \\ 0 & -\sin \eta_1 & \cos \eta_1 \end{pmatrix}, \quad (8)$$

$$a_2 = \begin{pmatrix} \cos \eta_2 & \sin \eta_2 & 0 \\ -\sin \eta_2 & \cos \eta_2 & 0 \\ 0 & 0 & e^{-i\chi_2} \end{pmatrix}, \quad (9)$$

where χ_1 and χ_2 are the mixing phases for active and sterile neutrinos, while η_1 and η_2 are their mixing angles. In our calculations, we will use the following test values for the new mixing parameters:

$$\chi_1 = \chi_2 = -\pi/2, \quad \eta_1 = 5^\circ, \quad \eta_2 = \pm 30^\circ. \quad (10)$$

In the third case, we specify the matrix a_3 in a diagonal form with two phases $\zeta_1 = \pi/2$ and $\zeta_2 = \pi/4$, so that

$$a_3 = \text{diag}\{e^{-i\zeta_1}, e^{-i\zeta_2}, 1\}. \quad (11)$$

In all three cases, we assume that the small parameter ϵ satisfies the condition $\epsilon \lesssim 0.03$.

For the active neutrinos, we will use the results obtained in Refs. [21, 22] on estimating the absolute values of the masses m_i ($i = 1, 2, 3$) of light active neutrinos for the NH case (in eV units):

$$m_1 \approx 0.0016, \quad m_2 \approx 0.0088, \quad m_3 \approx 0.0497. \quad (12)$$

We can now examine three versions of the generalized $(3+3)$ -model that correspond to the so-called $(3+1)$ - and $(3+2)$ -models of active and sterile neutrinos. As was indicated above, these models are used to explain the anomalies that manifest themselves in neutrino data at short distances.

In the first version of the model, which corresponds to the $(3+1)$ -model, we choose a value for the sterile-neutrino mass $m_{3'}$ in the region around 1 eV in accordance with the results obtained in Ref. [4] for the best value of the mass of only one sterile neutrino in this region: $m_{3'} \approx 0.96$ eV. We specify the masses $m_{2'}$ and $m_{1'}$ of the other two sterile neutrinos in the region around 30 eV by using the result obtained in Ref. [15] by fitting the mass of the warm-dark-matter particle. We assume that the particles of two components of warm dark matter have masses belonging to this range and taking values of $m_{2'} \approx 28$ eV and $m_{1'} \approx 32$ eV. Within this version, we take the matrix a_1 for the matrix a .

In the second version of our model, which corresponds to the $(3+2)$ -model, we choose values for the masses $m_{3'}$ and $m_{2'}$ of two sterile neutrinos in the region around 1 eV, following the result reported in Ref. [4] for the best values of the sterile-neutrino masses in the $(3+2)$ -model, that is, $m_{3'} \approx 0.69$ eV and $m_{2'} \approx 0.93$ eV. For the mass $m_{1'}$ of the third sterile neutrino, we make use of the mass of the possible dark-matter particle [40–42], that is, $m_{1'} \approx 7100$ eV; on the other hand, the choice of this value has virtually no effect on the results obtained below. In this version, we take the matrix a_2 for the matrix a .

In the third version, which we will call the $(3+1+1+1)$ -model, we choose values for the masses $m_{3'}$ and $m_{2'}$ of two sterile neutrinos in the region around 1 eV according to the results obtained in Ref. [4] for the smallest and the largest values of the possible sterile-neutrino masses, that is, $m_{3'} \approx 0.32$ eV and $m_{2'} \approx 3.15$ eV. We recall that these masses correspond to the boundaries of the interval of values admissible for the squares of sterile-neutrino masses, namely, $0.1 \text{ eV}^2 < m_s^2 < 10 \text{ eV}^2$. The mass of the third sterile neutrino, $m_{1'}$, is determined by the result obtained in Ref. [15] by fitting the mass of the warm-dark-matter particle, that is, $m_{1'} \approx 30$ eV. In this version, we take the matrix a_3 for the matrix a .

Thus, the absolute values of the masses of all neutrinos for three versions of the model (in eV units) can be represented in the form

$$m_{\nu,1} = \{0.0016, 0.0088, 0.0497, 0.96, 28, 32\}, \quad (13a)$$

$$m_{\nu,2} = \{0.0016, 0.0088, 0.0497, 0.69, 0.93, 7100\}, \quad (13b)$$

$$m_{\nu,3} = \{0.0016, 0.0088, 0.0497, 0.32, 3.15, 30\}. \quad (13c)$$

Taking into account these mass distributions of sterile neutrinos, we can call the above three versions of the $(3+3)$ neutrino model, respectively, as the $(3+1+2)$ -model, $(3+2+1)$ -model, and $(3+1+1+1)$ -model of active and sterile neutrinos. Note that these models can in principle mimic the $(3+1)$ - and $(3+2)$ -models if the contribution of one or two sterile neutrinos decreases, for example, upon a significant increase in their masses or upon a special choice of the mixing angles.

4. EQUATIONS FOR NEUTRINO PROBABILITY AMPLITUDES AND PROBABILITIES OF OSCILLATION TRANSITIONS BETWEEN ACTIVE AND STERILE NEUTRINOS

The equations for the flavor amplitudes of neutrino propagation in a vacuum for the case of three active neutrinos are well known (see, e.g., Ref. [23]) and are given by

$$i\partial_r \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}, \quad (14)$$

where the matrix H is expressed in terms of the Pontecorvo–Maki–Nakagawa–Sakata mixing matrix $U_{PMNS} \equiv U$ as

$$H = \frac{U}{2E} \begin{pmatrix} m_1^2 - m_0^2 & 0 & 0 \\ 0 & m_2^2 - m_0^2 & 0 \\ 0 & 0 & m_3^2 - m_0^2 \end{pmatrix} U^\dagger. \quad (15)$$

Here, m_0 is the lowest neutrino mass value among the three masses m_1 , m_2 , and m_3 , while E is the neutrino energy.

In the general case when the number of different neutrino flavors is $3+N$, the matrix Δ_{m^2} of the mass differences can be defined as

$$\Delta_{m^2} = \text{diag}\{m_1^2 - m_0^2, m_2^2 - m_0^2, \dots, m_{3+N}^2 - m_0^2\}, \quad (16)$$

where m_i ($i = 1, 2, \dots, 3+N$) are the neutrino masses and m_0 is the lowest neutrino mass among all m_i . In the case of the $(3+3)$ -model, we obtain the following equations for the neutrino propagation, which generalize Eqs. (14) and (15) and have the form

$$i\partial_r \begin{pmatrix} a_a \\ a_s \end{pmatrix} = \frac{\tilde{U}}{2E} \Delta_{m^2} \tilde{U}^\dagger \begin{pmatrix} a_a \\ a_s \end{pmatrix}, \quad (17)$$

where \tilde{U} is the unitary 6×6 mixing matrix for active and sterile neutrinos that is specified by Eq. (7). For antineutrinos, the respective equations have the form

$$i\partial_r \begin{pmatrix} a_{\bar{a}} \\ a_{\bar{s}} \end{pmatrix} = \frac{\tilde{U}^*}{2E} \Delta_{m^2} \tilde{U}^T \begin{pmatrix} a_{\bar{a}} \\ a_{\bar{s}} \end{pmatrix}, \quad (18)$$

where $*$ denotes complex conjugation. While solving these equations with taking into account the oscillation parameter values given in Sec. 3, we can find the survival

probabilities for electron or muon neutrinos or antineutrinos, as well as the appearance or disappearance probabilities for neutrinos or antineutrinos of any other flavor, versus the neutrino/antineutrino energy and versus the distance from the source.

From Eqs. (17) and (18) one can obtain analytical expressions for transitions between neutrino/antineutrino flavors in a vacuum versus the distance L from the source. If \tilde{U} is the generalized 6×6 mixing matrix of the form given by Eq. (7) and if we introduce the notation $\Delta_{ki} \equiv \Delta m_{ik}^2 L / (4E)$, then, according to Ref. [28], one can calculate the probabilities for the transitions of ν_α to $\nu_{\alpha'}$ or of $\bar{\nu}_\alpha$ to $\bar{\nu}_{\alpha'}$ by the formula

$$\begin{aligned}
P(\nu_\alpha(\bar{\nu}_\alpha) \rightarrow \nu_{\alpha'}(\bar{\nu}_{\alpha'})) &= \delta_{\alpha'\alpha} - \\
&- 4 \sum_{i>k} \text{Re}(\tilde{U}_{\alpha'i} \tilde{U}_{\alpha i}^* \tilde{U}_{\alpha'k}^* \tilde{U}_{\alpha k}) \sin^2 \Delta_{ki} \pm \\
&\pm 2 \sum_{i>k} \text{Im}(\tilde{U}_{\alpha'i} \tilde{U}_{\alpha i}^* \tilde{U}_{\alpha'k}^* \tilde{U}_{\alpha k}) \sin 2\Delta_{ki}, \quad (19)
\end{aligned}$$

where the upper sign (+) corresponds to the $\nu_\alpha \rightarrow \nu_{\alpha'}$ neutrino transitions, while the lower sign (−) corresponds to the $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$ antineutrino transitions. Note that the flavor indices α and α' (just as the summation indices i and k for massive states) refer to all neutrinos, both active and sterile ones. Moreover, it follows from Eq. (19) that the relation $P(\nu_\alpha \rightarrow \nu_{\alpha'}) \equiv P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'})$ holds exactly as a consequence of CPT conservation [28]. In order to control the accuracy of numerical results obtained on the basis of Eqs. (17) and (18) with allowance made for the fine sterile-neutrino effects, the calculations were also performed with using the exact analytical expressions (19).

5. NUMERICAL RESULTS FOR OSCILLATIONS OF ACTIVE AND STERILE NEUTRINOS

In the present study, we focus primarily on the possibility of describing, within the model versions being considered, the anomalies that were found in data from the LSND experiment on the oscillations of accelerator muon neutrinos and antineutrinos, which afterwards were tested and will be still tested in a number of future accelerator experiments [43]. This refers to data on the disappearance of muon neutrinos and antineutrinos and to the appearance of electron neutrinos and antineutrinos in the processes $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$. A typical ratio of the distance travelled by a neutrino before detection to the neutrino energy is either several meters per megaelectronvolt or one meter per several megaelectronvolts. Attempts at a simultaneous description of all data in these processes lead to difficulties. In particular, problems associated with different values of the excess of the ν_e and $\bar{\nu}_e$ yields in the MiniBooNE experiment can be solved upon admitting CP violation [44–46]. It should be noted that the reactor and gallium anomalies manifesting themselves in neutrino data as the disappearance of electron neutrinos and antineutrinos can be described within the

model being considered by appropriate choosing a value of the parameter ϵ . This leads to values smaller than unity for the parameter $\varkappa = 1 - \epsilon$ appearing in expression (7) and, hence, to a deficit of electron neutrinos and antineutrinos.

The appearance probabilities for electron neutrinos and antineutrinos in accelerator beams of muon neutrinos and antineutrinos are shown in Figs. 1 and 2, respectively, versus the distance from the source (from zero up to 100 m) at various values of the neutrino energy in the range between 20 and 50 MeV. Shown in the same figures for the sake of comparison are the dependences obtained for the analogous probabilities within the model containing only three active neutrinos, that is, in the absence of sterile-neutrino contributions (at $\epsilon = 0$).

In Fig. 1, the dependences of the appearance probabilities for electron (left-hand panels) neutrinos and (right-hand panels) antineutrinos in beams of muon neutrinos and antineutrinos, respectively, on the distance from the source are shown at various neutrino-beam energies and at various values of coupling constant ϵ of active and sterile neutrinos for two cases of mixing matrices, a_1 and a_3 . In these two cases, the results are found to be identical, and, as can be seen from Fig. 1, the effect of sterile neutrinos on the appearance probability for electron neutrinos/antineutrinos in these cases is negligibly small in the range of values up to $\epsilon = 0.01$ of the coupling constant of active and sterile neutrinos. At the ϵ -values considered here, the neutrino and antineutrino yields are nearly identical at neutrino energies of interest between 20 and 50 MeV. According to the results of our calculations, the distinction between the yields of electron neutrinos and antineutrinos for these mixing-matrix versions that is associated with a nonzero value of the phase δ_{CP} manifests itself at the distances under consideration only at neutrino energies not higher than 1 MeV and is weakly dependent on the parameter ϵ .

In Fig. 2, the dependences of the appearance probabilities for electron (left-hand panels) neutrinos and (right-hand panels) antineutrinos in beams of muon neutrinos and antineutrinos, respectively, on the distance from the source are shown at various neutrino-beam energies and various values of coupling constant ϵ of active and sterile neutrinos for the mixing matrix a_2 at the test parameter value of $\eta_2 = \pi/6$. One can see that this case differs drastically from the previous cases where the mixing matrix is a_1 or a_3 , the results for which are presented in Fig. 1 at the neutrino energies between 20 and 50 MeV. Firstly, the relative effect of sterile neutrinos is by and large substantially stronger in the current case and has the character of oscillations, as can be seen from the results for the neutrino energy of 20 MeV. Moreover, the relative neutrino and antineutrino yields at the parameter value of $\epsilon = 0.005$, for example, are approximately two orders of magnitude greater (about $\sim 10^{-4}$) than those at $\epsilon = 0$ (about $\sim 10^{-6}$). Secondly, the yields of electron neutrinos and antineutrinos are markedly different at this value of the parameter ϵ in the neutrino-energy

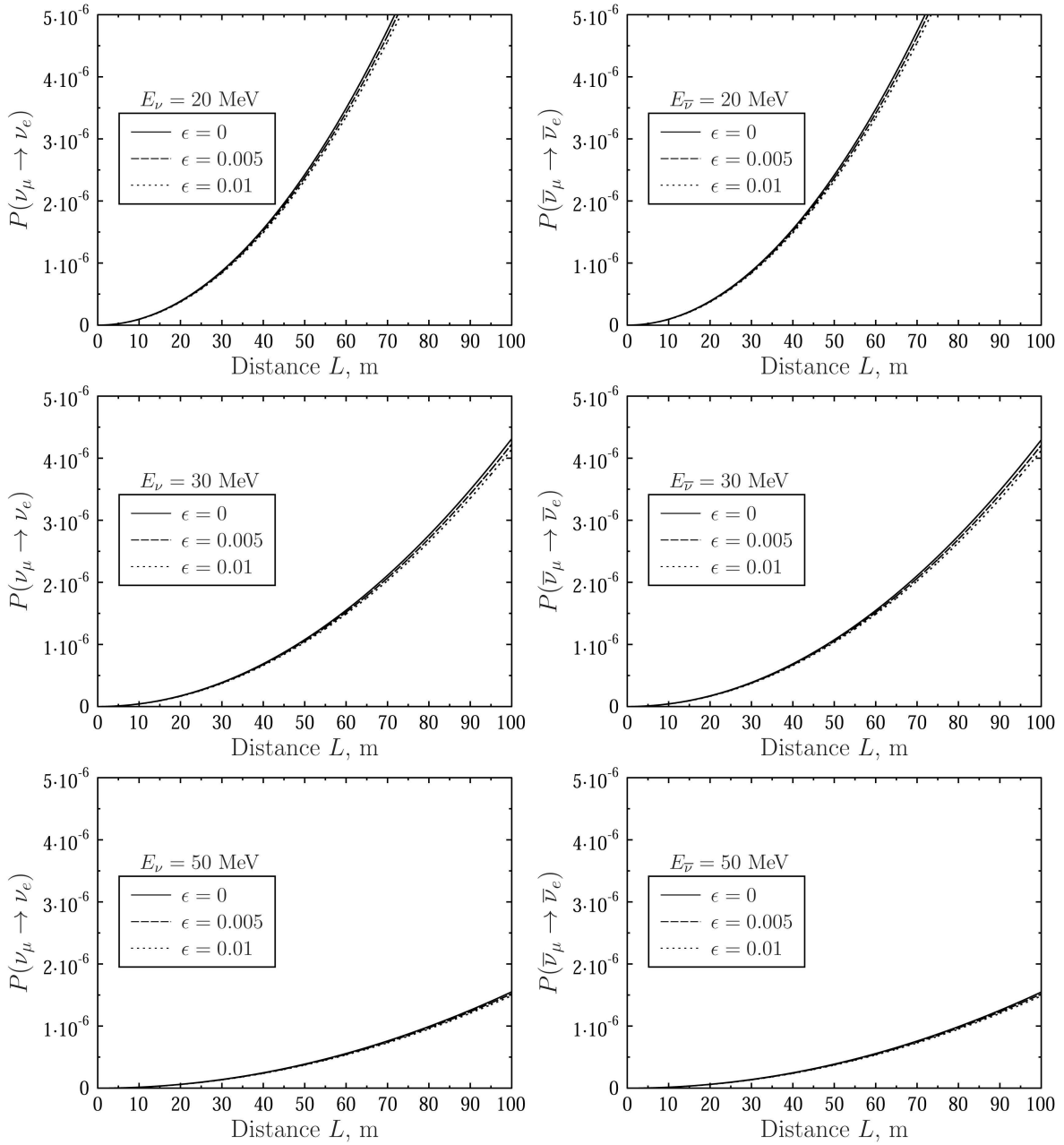


FIG. 1. Appearance probability for electron (left-hand panels) neutrinos and (right-hand panels) antineutrinos as a function of the distance from the source in beams of muon neutrinos and antineutrinos, respectively, at various neutrino-beam energies and various values of coupling constant ϵ of active and sterile neutrinos for two mixing matrices a_1 and a_3 leading to identical results.

range of $20 \div 50$ MeV that we consider. The appearance probabilities for electron antineutrinos and neutrinos are different owing to CP violation, but, without sterile neutrinos (that is, at $\epsilon = 0$), this difference is extremely small at neutrino energies in the range of $20 \div 50$ MeV. However, for the case of mixing matrix of the a_2 -type it becomes quite distinct upon taking into account sterile neutrinos at the above values of the parameter ϵ .

The physical reason of this difference between the different versions of the mixing matrix is that, for the ver-

sions where the mixing matrix is either a_1 or a_3 , the probability for the transition of a muon neutrino/antineutrino to an electron neutrino/antineutrino does not contain any contributions from sterile neutrinos, and in both cases in question these transition probabilities are the same and are determined only by the mixing parameters of active neutrinos. Mathematically, this is expressed by the fact that, in the analytic expression (19), only the contributions with $(i = 2, k = 1)$, $(i = 3, k = 1)$, and $(i = 3, k = 2)$ are nonzero in the sum over i and k at $\alpha = 2$

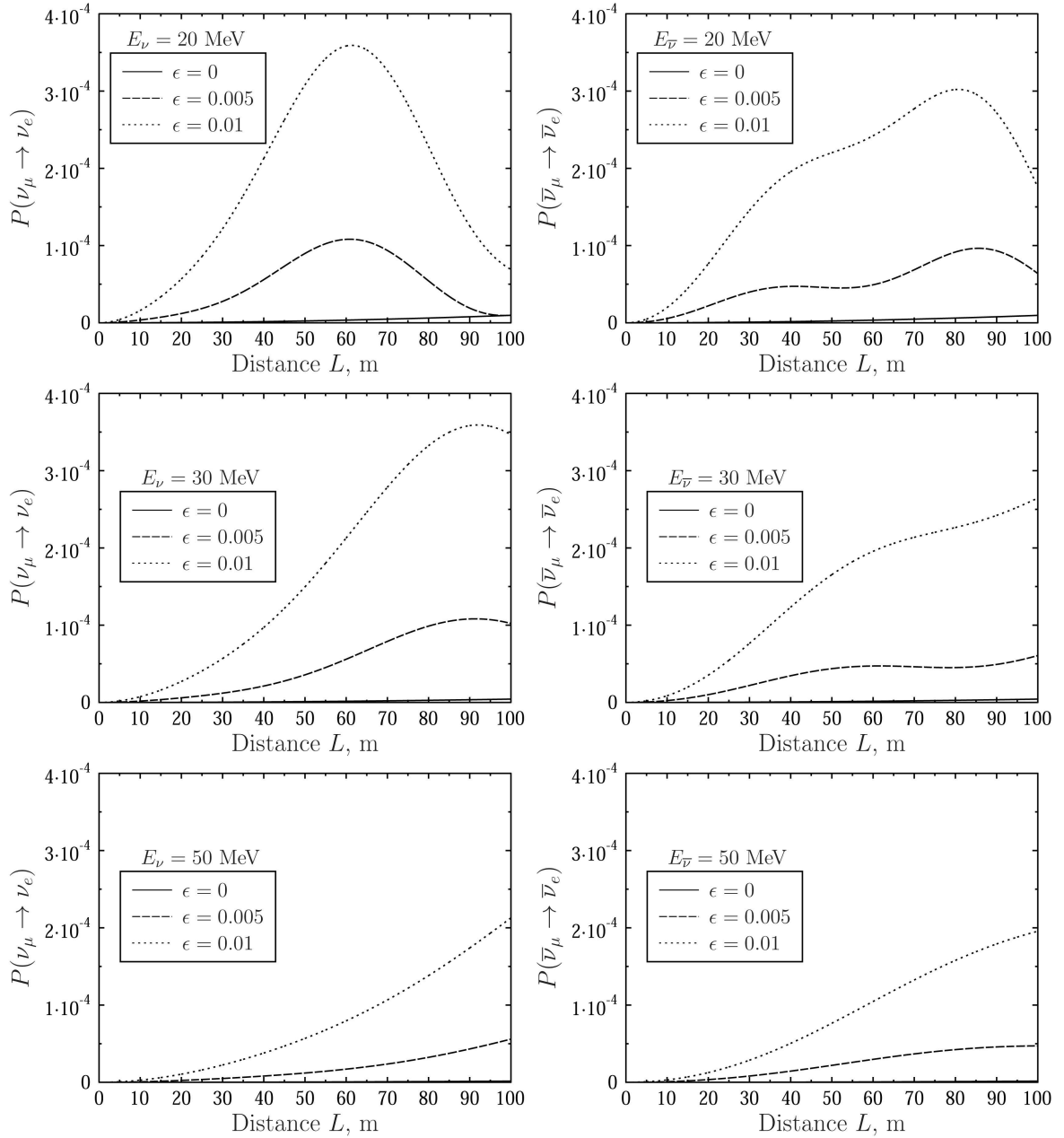


FIG. 2. Appearance probability for electron (left-hand panels) neutrinos and (right-hand panels) antineutrinos as a function of the distance from the source in beams of muon neutrinos and antineutrinos, respectively, at various neutrino-beam energies and various values of coupling constant ϵ of active and sterile neutrinos for the case of mixing matrix a_2 at $\eta_2 = \pi/6$.

and $\alpha' = 1$. All other contributions vanish identically for mixing matrices of the a_1 or a_3 type. But in the case of a mixing matrix belonging to the a_2 type, the terms with $(i = 4, k = 1)$, $(i = 4, k = 2)$, $(i = 4, k = 3)$, $(i = 5, k = 1)$, $(i = 5, k = 2)$, $(i = 5, k = 3)$, and $(i = 5, k = 4)$ result in nonzero contributions in the sum over i and k , in addition to the aforementioned nonzero contributions. At the same time, the contributions with $i = 6$ and $k < i$ again vanish identically. Nevertheless, the mixing between all active neutrinos and two light sterile neutrinos in the case of the mixing matrix a_2 pro-

vides a significant contribution to the probabilities for the transitions of a muon neutrino/antineutrino to an electron neutrino/antineutrino, and this contribution proves to be large owing primarily to the term with $(i = 5, k = 4)$, which has the same values for the probabilities of the muon-neutrino transition to an electron neutrino and the muon-antineutrino transition to an electron antineutrino. At the same time, the difference between the probabilities for the muon-neutrino transition to an electron neutrino and the muon-antineutrino transition to an electron antineutrino is determined by the terms with

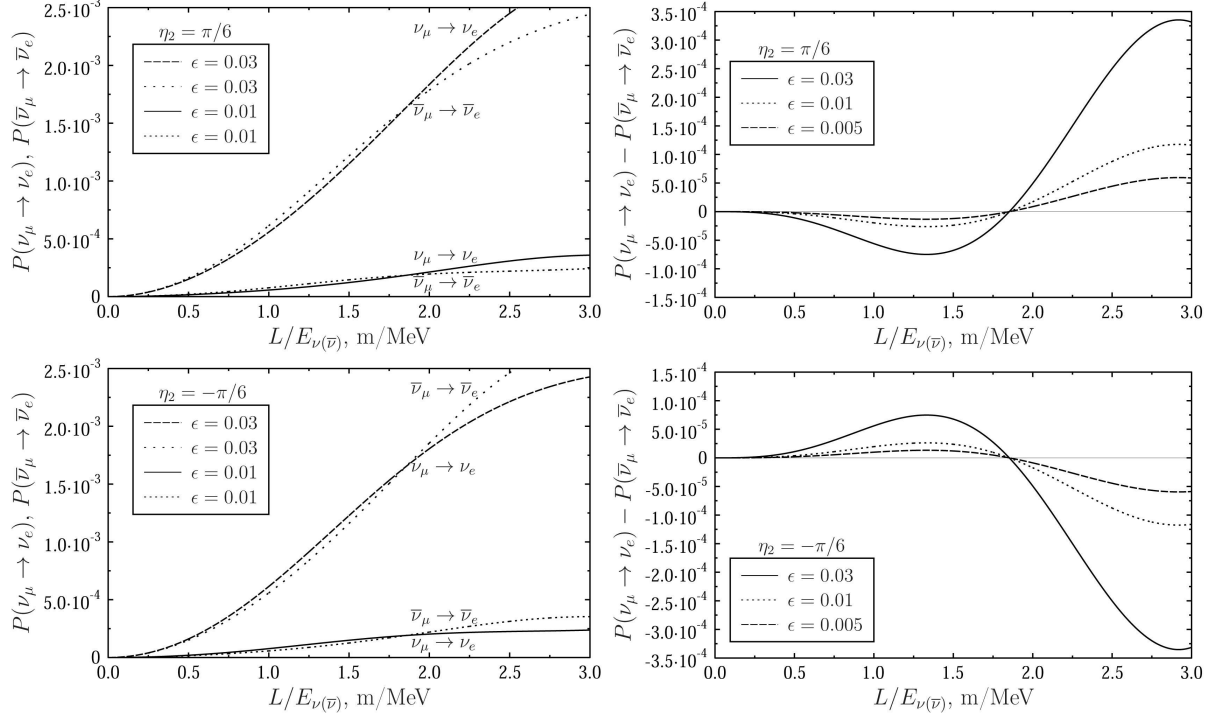


FIG. 3. Appearance probabilities for electron neutrinos and antineutrinos in beams of muon neutrinos and antineutrinos, respectively, versus the ratio of the distance from the source to the neutrino energy at various values of coupling constant ϵ of active and sterile neutrino for the case of mixing matrix a_2 (left-hand panels) and difference of these probabilities (right-hand panels) at $\eta_2 = \pi/6$ (upper panels) and $\eta_2 = -\pi/6$ (lower panels).

($i = 4, k = 1$), ($i = 4, k = 2$), ($i = 4, k = 3$), ($i = 5, k = 1$), ($i = 5, k = 2$), and ($i = 5, k = 3$), which are different for these transitions, the terms with $k = 3$ being dominant here because of a larger value of the mass m_3 . As can be seen from Figs. 2–5, the values of these contributions increase as the value of the parameter ϵ becomes larger.

Thus, a significant increase in the probabilities for the transitions of muon neutrinos/antineutrinos to electron neutrinos/antineutrinos is determined by the choice of the matrix a_2 and by the masses of light sterile neutrinos. At the same time, this depends only slightly on the active-neutrino masses as long as their values are substantially lower than the masses of light sterile neutrinos and is virtually independent of the mass of the heavy sterile neutrino. However, the largest mass among the active neutrinos (m_3) has a substantial influence on the yield asymmetry, that is, on the difference in the neutrino and antineutrino yields. Note that, in order to demonstrate the effect, we have taken the most typical test values for the model parameters, which are compatible with experimental data and with the results of astrophysical observations, and verified that no qualitative changes arise in the results upon varying these parameters, so that our result is stable in this sense.

In the following, we restrict ourselves to studying mixing matrices of just the a_2 type, since only in this case the results for the probabilities of appearance of electron

neutrinos/antineutrinos may correspond to the anomaly noticed in the LSND experiment at an energy of about 40 MeV and a distance of about 30 m (see Figs. 1 and 2). Figure 2 shows that the probabilities for the appearance of electron neutrinos/antineutrinos in the range between 10 and 30 m vary substantially, which may relate to a negative result in searches for the LSND anomaly in the KARMEN experiment [47, 48], which was performed at a distance of about 17.5 m. It can be seen from Fig. 2 that a positive result in experiments of this type can be obtained only at high neutrino fluxes and over long experimental times, since the transition probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, as well as the absolute values of their difference, are not larger than 10^{-2} and depend on the parameter ϵ , which should not exceed at least 0.1.

On the left-hand panels of Fig. 3, the appearance probabilities of electron neutrinos and antineutrinos in beams of muon neutrinos and antineutrinos, respectively, at various coupling constants ϵ of active and sterile neutrinos for the case of the mixing matrix a_2 are shown versus the ratio of the distance L from the source to the energy $E_{\nu(\bar{\nu})}$ of neutrino/antineutrino beams at two values of the parameter η_2 , namely, $\eta_2 = +\pi/6$ (upper panels) and $\eta_2 = -\pi/6$ (lower panels). The graphs representing the resulting difference of the appearance probabilities for electron neutrinos and antineutrinos (asymmetry) are exhibited on the right-hand panels of Fig. 3. An important result, as is seen from Fig. 3, is the change of the

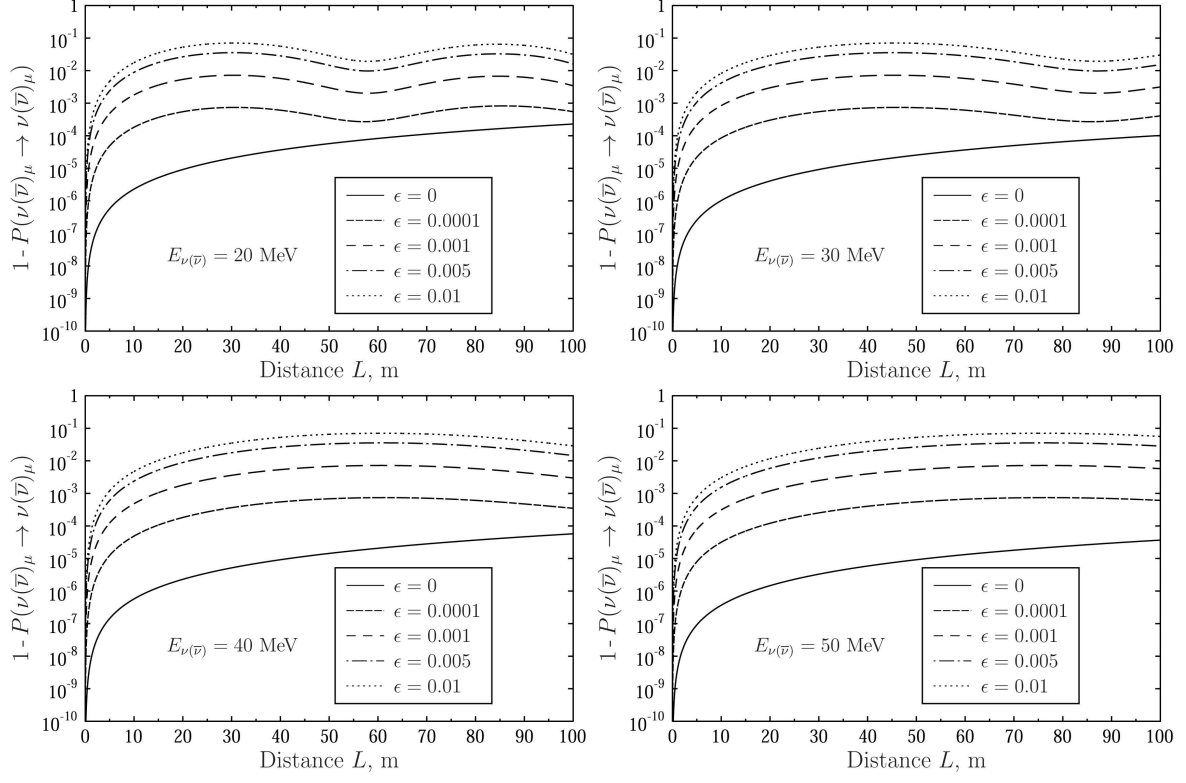


FIG. 4. Disappearance probability for muon neutrinos (antineutrinos) as a function of the distance from the source in beams of muon neutrinos (antineutrinos) at various energies of neutrino beams and various values of coupling constant ϵ of active and sterile neutrinos for the case of mixing matrix a_2 .

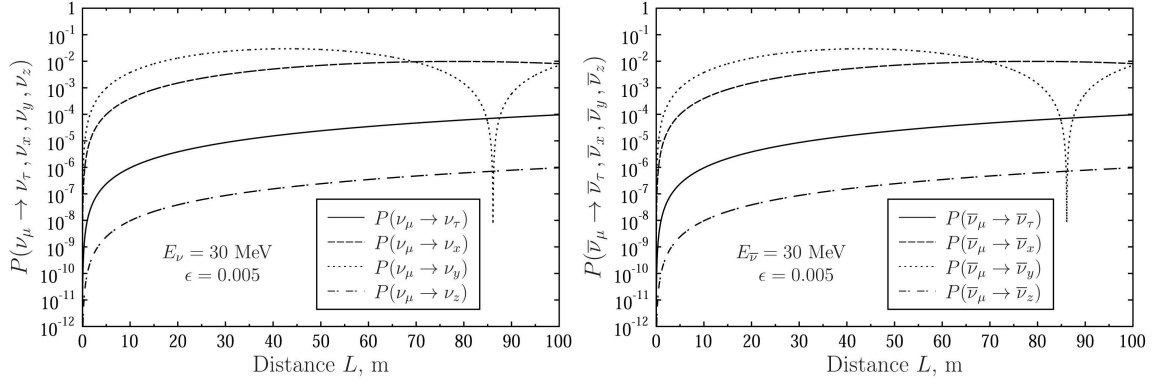


FIG. 5. Appearance probabilities for tau and three sterile (left-hand panel) neutrinos and (right-hand panel) antineutrinos versus the distance from the source in beams of muon neutrinos and antineutrinos, respectively, at the neutrino-beam energy 30 MeV and at the value of $\epsilon = 0.005$ of coupling constant between active and sterile neutrinos for the case of mixing matrix a_2 .

sign of this difference when values of $L/E_{\nu(\bar{\nu})} \approx 2$, so that at short distances (more precisely, distances shorter than 60 m at the energy of $E_{\nu(\bar{\nu})} = 30$ MeV) the antineutrino yield is higher within our model than the neutrino yield, but that at rather long distances the neutrino yield should exceed the antineutrino yield, if $\eta_2 = +\pi/6$. We emphasize that the sign of this asymmetry changes in response to reversal of the η_2 sign (even though it is not

a strictly odd function) and depends on the parameter $L/E_{\nu(\bar{\nu})}$. Besides, the sign of the asymmetry changes upon going over from $\delta_{CP} = -\pi/2$ to $\delta_{CP} = \pi/2$. On the left-hand panels of Fig. 3, the results for $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ are shown in the same form and versus the ratio $L/E_{\nu(\bar{\nu})}$ as it was done for the results of the LSND and MiniBooNE experiments (see Fig. 3 in Ref. [49]). A comparison of these figures indicates that,

within the model being considered, it is possible to explain the appearance of the accelerator anomaly in the data from those experiments.

The disappearance probability for muon neutrinos [according to Eq. (19), it coincides with the disappearance probability for muon antineutrinos] in beams of muon neutrinos (antineutrinos) is shown in Fig. 4 as a function of the distance from the source at various neutrino-beam energies and various values of coupling constant ϵ of active and sterile neutrinos for the mixing matrix a_2 . It is seen that with the neutrino beam energy variation the effect of sterile neutrinos to this probability behaves qualitatively in the same fashion as in the case with the appearance probability of electron neutrinos. That is, this probability grows monotonically as the effect of sterile neutrinos becomes stronger.

In Fig. 5, the appearance probabilities for tau and three sterile (left-hand panel) neutrinos and (right-hand panel) antineutrinos in beams of muon neutrinos and antineutrinos, respectively, are shown versus the distance from the source at the neutrino-beam energy of 30 MeV and at the value of $\epsilon = 0.005$ of coupling constant between active and sterile neutrinos for the mixing matrix a_2 . One can see that the appearance of two sterile neutrinos (x - and y -neutrinos) has the highest probability, while the appearance probabilities for a tau active neutrino and a sterile z -neutrino are relatively small. Formally, there is a difference here between the appearance probabilities of neutrinos and antineutrinos, as this must be by virtue of normalization conservation, on one hand, and the difference between the appearance probabilities for electron neutrinos and antineutrinos, on the other hand. However, the difference between the appearance probabilities for x - and y -neutrinos and corresponding antineutrinos is relatively very small because of the smallness of the appearance probabilities for electron neutrinos and antineutrinos and is visually unobservable.

All these results are a feature peculiar to the above versions of the model of active and sterile neutrinos, and they can be used in interpreting available experimental data and in predicting the results of new relevant experiments.

6. CONCLUSION

Available experimental data and proposed theoretical models indicate that neutrinos have unusual properties, and further intensive theoretical and experimental studies are necessary to explain these properties. At the present time, it is of greatest interest to test the existence of light sterile neutrinos and, in the positive case, to determine their number and to fix with a pinpoint accuracy the absolute values of both active- and sterile-neutrino masses.

In this paper, the properties of active and sterile neutrinos versus the sterile-neutrino masses and versus the form of the mixing matrix for active and sterile neutrinos have been examined on the basis of a phenomenological

neutrino model with three active and three sterile neutrinos. Specifically, this has been done by using three versions of this model, namely, the $(3+1+2)$ -, $(3+2+1)$ -, and $(3+1+1+1)$ -model. In this, a new parametrization of a general mixing matrix for active and sterile neutrinos has been used, which makes it possible to take into account the sterile-neutrino effects quite easily. The properties of oscillations of active and sterile neutrinos in a vacuum at test values of model parameters have been calculated. All calculations have been performed for the case of a normal hierarchy of the mass spectrum of active neutrinos, taking into account possible CP violation in the lepton sector and setting the Dirac CP -phase in the U_{PMNS} -matrix for active neutrinos to $-\pi/2$. The dependences of the survival probabilities for muon neutrinos and antineutrinos and the probabilities for their transitions to electron neutrinos and antineutrinos on the distance from the source and on the ratio of this distance to the neutrino energy have been determined for neutrino energies not higher than 50 MeV.

The $(3+1+2)$, $(3+2+1)$, and $(3+1+1+1)$ versions of the general $(3+3)$ -model have been considered in order to interpret experimental neutrino oscillation data permitting the existence of the LNSD anomaly at a neutrino energy of about 40 MeV and a distance of about 30 m from the source. Graphs representing the survival probabilities for muon neutrinos and antineutrinos and the probabilities for their transitions to electron neutrinos and antineutrinos have been plotted versus admissible values of the model parameters. It has been shown (see Figs. 2 and 3) that this anomaly can be described within the $(3+2+1)$ -version of the model unlike the other two versions. This version that has been found is characterized by a specific mixing matrix for active and sterile neutrinos (namely, the matrix of the a_2 type), the Dirac CP phase of $\delta_{CP} = -\pi/2$, a normal hierarchy in the mass spectrum of active neutrinos, and a specific mass spectrum of sterile neutrinos that includes two masses of about 1 eV and one mass of about 7 keV. The results for this case on the appearance probabilities for electron neutrinos and antineutrinos may correspond to the anomaly observed in the LSND experiment. The model in question may also be used to describe some astrophysical data for the cases where sterile neutrinos or warm-dark-matter particles are involved. As for known cosmological constraints on the total number of relativistic neutrinos [24, 25], it should be noted that these constraints are model-dependent. Moreover, there are the models even at the present time, in the framework of which one can moderate substantially or even remove these constraints, assuming different conditions of interaction and thermal equilibrium between active and sterile neutrinos [26, 27]. Thus, there is the possibility of removing the possible inconsistency of the model being considered, which contains three active and three sterile neutrinos, with the data obtained from the cosmological observations.

It is planned to carry out in the near future a number of ground-based experiments aimed at searches for light

sterile neutrinos [1, 5, 16–18]. For example, searches for light sterile neutrinos have been proposed in the experiments with high fluxes of electron antineutrinos arising in the process of β -decay of a large number of ^8Li nuclei [50]. Besides, the realization of two accelerator experiments whose results would make it possible to resolve, with a high degree of reliability, the problem of the LSND anomaly is being prepared. These are the OscSNS experiment in United States of America [51] and J-PARC MLF

experiment in Japan [52] at neutrino energies around 40 MeV with detectors positioned at a distance from the neutrino source in the range from 10 up to 100 m. The results obtained in the present study for the oscillation properties of active and sterile neutrinos can be used, in particular, for interpreting the results of these and similar experiments.

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